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# IS THE FEAR OF INTEREST RATE VOLATILITY OVERRATED? A SIMULATION CASE STUDY OF BONDHOLDERS' LONG-TERM RETURNS

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## ABSTRACT

*In this article we propose a simple simulation case study with a specific pedagogical objective. Finance students often struggle to fully understand the difference between coupon rate and yield to maturity; they also seem to have difficulty comprehending the reinvestment rate assumption embedded in the bond pricing model. The simulation case discussed in this paper helps students attain a deeper understating of the reinvestment rate assumption and yield to maturity that would help them not only with cognizance of bonds underlying arithmetic but also with comprehension of related topics such as internal rate of return (IRR) in capital budgeting projects. We simulate cash in/outflows of an investor that makes fixed periodic contributions to her one-bond portfolio and liquidates her position after a decade (or a five-year period with semiannual coupon payments). Results suggest that long-term bond investors should not be unduly concerned with interest rate fluctuations for two reasons: first, the dollar cost-averaging nature of steady contributions mitigates the downside risk; second, the opportunity to reinvest 'guaranteed' interest payments back in the portfolio helps in achieving higher holding period returns. Our simulation results indicate that long-term bond investors would have earned positive returns exceeding the rate of inflation, with the exception of 1971-81 period, regardless of the direction that the yield took in any particular decade. The 1971-81 period however was concurrent with extreme rise in the yield. In July 1981 the yield reached the all-time high of 15.84%. We show that in this decade the inflation-adjusted return is equal to -0.27% which is a small loss in magnitude compared to the gains in other decades.*

***Keywords:*** *Interest Rate Volatility, Bondholders, Long-term Returns, Federal Reserve.*

## 1. Introduction

Bond investors are flooded with doomsday reports every time Federal Reserve signals rising interest rates. However, “if history is any guide, the disease of rising interest rates will only have a mild impact on bond returns—as rates rise, bond prices decline. And seeking immunity from any losses by buying riskier assets could lead to trouble, analysts say.” according to an article in The Wall Street Journal. The article states:

> *Rising rates also could have a positive impact on the income of many retirees, some of whom are at risk of running out of money in the long run, according to an analysis conducted for The Wall Street Journal by the Employee Benefit Research Institute. That is because while a steep interest-rate rise will cost bond investors in the near term, they will gain back their losses and more over time as they buy new bonds at higher interest rates.*

In this paper we propose a simulation case study designed to replicate a simple long-term investment strategy in a simple bond portfolio. Results of the simulation show that long-term bond investors should not be too concerned with interest rate volatility due to the fact that regular contributions to bond portfolios (such as contributions made to bond funds in IRA and 401k plans) can considerably lessen the downside risk. In fact, the primary benefit of such dollar cost-averaging strategy is the opportunity it provides for investors to reinvest “guaranteed” interest payments back in the portfolio which boosts their long-term returns as interest rates rise.

We pursue two main objectives: first, the case studied in this paper is intended to have an in-class application. Finance students often struggle to fully comprehend the difference between coupon rate and yield to maturity. They also seem to have a hard time understanding the reinvestment rate assumption embedded in bond pricing. Asking students to do similar simulations could greatly help them attain a deeper understanding of bonds and the underlying assumptions made when pricing bonds. Second, by simulating the buy-and-hold returns for a simple case of one-bond portfolio we show that long-term bond investors have earned descent returns regardless of the direction the yield has taken in different periods. We later show that, out of the four decades under study, only the 1972-82 decade holding period returns lagged the inflation rate.

### *1.1 Duration Model and Its Shortcomings*

All fixed income securities are subject to immediate drop in value as a result of a rise in the level of interest rate. This is also true for stocks since both stocks and bond markets face cash outflows as capital moves from these securities to safer investments such as money market securities when interest rates rise. As for bond portfolios, interest rate shock and duration are mainly the cause of fluctuations in the value. According to the Duration Model, the percentage change in a bond’s value for an annual bond is equal to:

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta r_b}{1 + r_b}\right]$$

And for a semiannual bond:

$$\frac{\Delta P}{P} = -D\left[\frac{\Delta r_b}{1 + r_b/2}\right]$$

The left side is the percentage change in the bond value, D is duration of a bond or a bond portfolio, and $r_b$ is the level of interest rate. Change in interest rate is shown by $\Delta r_b$. The model provides investors with a quick way of assessing the change in price as the yield fluctuates. For example, for a bond with duration of 5 years, the model predicts approximately 5% increase (decrease) in the value for 1% fall (rise) in the yield. But, the model has some serious limitations: first, due to the convex nature of the relationship between interest rate and percentage change in value, the estimations are only accurate for small changes in interest rate. Second, the model is shortsighted because it only reflects the immediate fall in bond values; thus, it fails to capture long-term buy and hold returns. In the next section we propose a simulation case study which shows a simple buy- and-hold strategy with or without regular contributions leads to significant holding period returns that far supersedes the historical inflation rate in all the decades studied in this paper, with the exception of 1972-82 period.

## 2. Case Study

### *2.1 Description*

For simplicity, in the case we present in this paper, we consider a portfolio of only one asset: U.S. 10 year-to maturity T-notes. We calculate holding-period returns defined as modified internal rate of return (MIRR hereafter). We estimate the MIRR earned by a long-term investor that invests $1000 a year for 10 years. The investor starts with $1000 investment and repeats this process 9 more times for a total of $10,000 invested. She then liquidates her position at the end of 10-year period. We simulate cash in/out flows of this simple investment strategy and calculate MIRRs for each scenario. We assume that the investor reinvests the coupon payments at the market imposed yield to maturity immediately after she receives the payment.

Figure 1 shows the yield on 10-year T-notes from 1962 to 2012.

**Figure 1: 10-Year T-note Yields from Yahoo Finance**


*Source: Yahoo Finance*

Table 1 shows T-Note yields for January 1962 to January 2012. The last column shows the average annual changes in the yield. The maximum rise (8.06%) followed by a subsequent deepest fall (-6.64%) in the yield occurred in 1972-82 and 1982-92 periods.

**Table 1:** Yield on 10-year Treasury Notes, 1962-2012

| Year | T-Notes Rate (%) | Change per Decade (%) | Average Change per Annum (%) |
|---|---|---|---|
| 1962 | 3.86 | | |
| 1972 | 6.12 | 2.26 | 0.2260 |
| 1982 | 14.18 | 8.06 | 0.8060 |
| 1992 | 7.54 | -6.64 | -0.6640 |
| 2002 | 5.41 | -2.13 | -0.2130 |
| 2012 | 2.22 | -3.19 | -0.3190 |

### *2.2 Assumptions and Limitations*

- Our investor contributes $1000 a year for a decade in a one-asset portfolio of 10-year to maturity T-notes. She liquidates her position at the end of the 10-year period. For simplicity, we only consider 10 year to maturity bonds; most investors however invest in a portfolio of bonds with different durations.
- Bond portfolio managers constantly rebalance and change their positions. This is not the case here. Our hypothetical investor simply follows a buy and hold strategy with fixed periodical contributions.
- We assume no callibility provision. While this is the correct assumption for T-notes, many corporate bonds may be called prior to maturity. This requires portfolio managers to factor in callability when dealing with corporate bonds.
- We assume annual coupon payments. This is not the case for T-notes or the majority of corporate bonds. T-notes have semiannual coupon payments. However, assuming annual coupon payments would only lead to the underestimation of holding period gains. Despite this we find significant MIRRs for this strategy for three out of four decades. In addition, we calculate MIRRs for two five-year periods of extreme change in the yield assuming semiannual compounding. Results are consistent.
- It is also assumed that the investor reinvests annual coupon payments as soon as she receives them.
- Finally, we assume that each year the yield increases or decreases by an equal amount (equal to the yield at the decade's end minus the yield at the beginning of the decade divided by ten). It will be more precise to use the actual asked yields instead.

### *2.3 Results*

In Tables 2 to 6 we simulate a simple investment strategy for five consecutive decades beginning with 1962 and calculate holding period returns (MIRR) for each decade[1]. We assume that the investor automatically reinvests the coupon payments at the market rate (T-notes yield to maturity) as soon as the coupon payments are remitted. The incremental annual change in the yield to maturity (YTM) is defined as:

$$\Delta YTM = (YTM_{10}-YTM_{0})/10$$

As stated before, results would be more accurate if the actual yields are used. Table 2 simulates the holding period return for our investor that invests $1000 in 10-year T-notes in January 1962 at 3.86%[2]. The investor continues to invest $1000 a year for 10 consecutive years. In addition, she reinvests the annual coupon payments as soon as she receives them. For simplicity, we assume annual payments instead of semiannual.

[1] In this paper MIRR and holding period return have identical meanings.
[2] Note that we use $1,000 for simplicity of use. The MIRR would not change for any other dollar of fixed contributions.

The last cell in each column shows the total amount invested each year which comprises a fresh $1000 contribution plus the sum of all coupon payments received from the bonds purchased in prior years. The last $1000 contribution occurs at the end of year 9. Up to that point the investor will have invested a total of $10,000 in T-notes. She finally liquidates her position at the end of year 10; at that point the total value of the bond portfolio will be equal to $12,500.69. It is important to remind students that annual contributions are made at the current market rate (yield to maturity) at the time when the bond is purchased. In order to estimate MIRR we calculate the present value of all contributions and add them up. The result is $8,216.66 that is the present value the total amount invested ($10,000). It is important to note that the discount rate (yield to maturity) is not fixed but changes every year. For example, the $1,000 contributed in year 2 is first discounted by 4.31% and then by 4.09%. Finally, we calculate MIRR (FV=$12,500.69, PV=- $8,216.66, N=10, PMT=$0 => MIRR=4.29%).

**Table 2:** Annual Contributions of $1000 to Treasury Notes (January 1962- January 1972 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 3.860% | | | | | | | | | | |
| Incremental Change | 0.226% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 3.86% | 4.09% | 4.31% | 4.54% | 4.76% | 4.99% | 5.22% | 5.44% | 5.67% | 5.89% | 6.12% |
| Investments | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | |
| | | $ 38.60 | $ 38.60 | $ 38.60 | $ 38.60 | $ 38.60 | $ 38.60 | $ 38.60 | $ 38.60 | $ 38.60 | $ 38.60 |
| | | $(1,038.60) | $ 42.44 | $ 42.44 | $ 42.44 | $ 42.44 | $ 42.44 | $ 42.44 | $ 42.44 | $ 42.44 | $ 42.44 |
| | | | $(1,081.04) | $ 46.61 | $ 46.61 | $ 46.61 | $ 46.61 | $ 46.61 | $ 46.61 | $ 46.61 | $ 46.61 |
| | | | | $(1,127.65) | $ 51.17 | $ 51.17 | $ 51.17 | $ 51.17 | $ 51.17 | $ 51.17 | $ 51.17 |
| | | | | | $(1,178.82) | $ 56.16 | $ 56.16 | $ 56.16 | $ 56.16 | $ 56.16 | $ 56.16 |
| | | | | | | $(1,234.98) | $ 61.63 | $ 61.63 | $ 61.63 | $ 61.63 | $ 61.63 |
| | | | | | | | $(1,296.61) | $ 67.63 | $ 67.63 | $ 67.63 | $ 67.63 |
| | | | | | | | | $(1,364.24) | $ 74.24 | $ 74.24 | $ 74.24 |
| | | | | | | | | | $(1,438.48) | $ 81.53 | $ 81.53 |
| | | | | | | | | | | $(1,520.02) | $ 89.59 |
| | | | | | | | | | | | $11,891.09 |
| PV, FV | $(8,216.66) | | | | | | | | | | $12,500.69 |
| MIRR | 4.29% | | | | | | | | | | |

Tables 3 to 6 use the exact same process to estimate MIRRs for the remaining 3 decades, 1972-2012.

**Table 3:** Annual Contributions of $1000 to Treasury Notes (January 1972- January 1982 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 6.12% | | | | | | | | | | |
| Incremental Change | 0.806% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 6.12% | 6.93% | 7.73% | 8.54% | 9.34% | 10.15% | 10.96% | 11.76% | 12.57% | 13.37% | 14.18% |
| Investments | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | |
| | | $ 61.20 | $ 61.20 | $ 61.20 | $ 61.20 | $ 61.20 | $ 61.20 | $ 61.20 | $ 61.20 | $ 61.20 | $ 61.20 |
| | | $(1,061.20) | $ 73.50 | $ 73.50 | $ 73.50 | $ 73.50 | $ 73.50 | $ 73.50 | $ 73.50 | $ 73.50 | $ 73.50 |
| | | | $(1,134.70) | $ 87.73 | $ 87.73 | $ 87.73 | $ 87.73 | $ 87.73 | $ 87.73 | $ 87.73 | $ 87.73 |
| | | | | $(1,222.43) | $ 104.37 | $ 104.37 | $ 104.37 | $ 104.37 | $ 104.37 | $ 104.37 | $ 104.37 |
| | | | | | $(1,326.80) | $ 123.98 | $ 123.98 | $ 123.98 | $ 123.98 | $ 123.98 | $ 123.98 |
| | | | | | | $(1,450.78) | $ 147.25 | $ 147.25 | $ 147.25 | $ 147.25 | $ 147.25 |
| | | | | | | | $(1,598.04) | $ 175.08 | $ 175.08 | $ 175.08 | $ 175.08 |
| | | | | | | | | $(1,773.12) | $ 208.55 | $ 208.55 | $ 208.55 |
| | | | | | | | | | $(1,981.67) | $ 249.06 | $ 249.06 |
| | | | | | | | | | | $(2,230.73) | $ 298.34 |
| | | | | | | | | | | | $13,428.22 |
| PV, FV | $(7,028.00) | | | | | | | | | | $14,957.29 |
| MIRR | 7.85% | | | | | | | | | | |

**Table 4**: Annual Contributions of $1000 to Treasury Notes (January 1982- January 1992 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 14.18% | | | | | | | | | | |
| Incremental Change | -0.664% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 14.18% | 13.52% | 12.85% | 12.19% | 11.52% | 10.86% | 10.20% | 9.53% | 8.87% | 8.20% | 7.54% |
| Investments | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | |
| | | $ 141.80 | $ 141.80 | $ 141.80 | $ 141.80 | $ 141.80 | $ 141.80 | $ 141.80 | $ 141.80 | $ 141.80 | $ 141.80 |
| | | $(1,141.80) | $ 154.33 | $ 154.33 | $ 154.33 | $ 154.33 | $ 154.33 | $ 154.33 | $ 154.33 | $ 154.33 | $ 154.33 |
| | | | $(1,296.13) | $ 166.58 | $ 166.58 | $ 166.58 | $ 166.58 | $ 166.58 | $ 166.58 | $ 166.58 | $ 166.58 |
| | | | | $(1,462.70) | $ 178.27 | $ 178.27 | $ 178.27 | $ 178.27 | $ 178.27 | $ 178.27 | $ 178.27 |
| | | | | | $(1,640.98) | $ 189.11 | $ 189.11 | $ 189.11 | $ 189.11 | $ 189.11 | $ 189.11 |
| | | | | | | $(1,830.08) | $ 198.75 | $ 198.75 | $ 198.75 | $ 198.75 | $ 198.75 |
| | | | | | | | $(2,028.83) | $ 206.86 | $ 206.86 | $ 206.86 | $ 206.86 |
| | | | | | | | | $(2,235.69) | $ 213.11 | $ 213.11 | $ 213.11 |
| | | | | | | | | | $(2,448.80) | $ 217.16 | $ 217.16 |
| | | | | | | | | | | $(2,665.96) | $ 218.72 |
| | | | | | | | | | | | $19,370.08 |
| PV, FV | $(6,345.13) | | | | | | | | | | $21,254.75 |
| MIRR | 12.85% | | | | | | | | | | |

**Table 5:** Annual Contributions of $1000 to Treasury Notes (January 1992- January 2002 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 7.54% | | | | | | | | | | |
| Incremental Change | -0.213% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 7.54% | 7.33% | 7.11% | 6.90% | 6.69% | 6.48% | 6.26% | 6.05% | 5.84% | 5.62% | 5.41% |
| Investments | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | |
| | | $ 75.40 | $ 75.40 | $ 75.40 | $ 75.40 | $ 75.40 | $ 75.40 | $ 75.40 | $ 75.40 | $ 75.40 | $ 75.40 |
| | | $(1,075.40) | $ 78.79 | $ 78.79 | $ 78.79 | $ 78.79 | $ 78.79 | $ 78.79 | $ 78.79 | $ 78.79 | $ 78.79 |
| | | | $(1,154.19) | $ 82.11 | $ 82.11 | $ 82.11 | $ 82.11 | $ 82.11 | $ 82.11 | $ 82.11 | $ 82.11 |
| | | | | $(1,236.30) | $ 85.32 | $ 85.32 | $ 85.32 | $ 85.32 | $ 85.32 | $ 85.32 | $ 85.32 |
| | | | | | $(1,321.62) | $ 88.39 | $ 88.39 | $ 88.39 | $ 88.39 | $ 88.39 | $ 88.39 |
| | | | | | | $(1,410.01) | $ 91.30 | $ 91.30 | $ 91.30 | $ 91.30 | $ 91.30 |
| | | | | | | | $(1,501.31) | $ 94.01 | $ 94.01 | $ 94.01 | $ 94.01 |
| | | | | | | | | $(1,595.32) | $ 96.50 | $ 96.50 | $ 96.50 |
| | | | | | | | | | $(1,691.82) | $ 98.73 | $ 98.73 |
| | | | | | | | | | | $(1,790.56) | $ 100.68 |
| | | | | | | | | | | | $14,200.52 |
| PV, FV | $(7,573.24) | | | | | | | | | | $15,091.76 |
| MIRR | 7.14% | | | | | | | | | | |

**Table 6:** Annual Contributions of $1000 to Treasury Notes (January 2002- January 2012 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 5.41% | | | | | | | | | | |
| Incremental Change | -0.319% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 5.41% | 5.09% | 4.77% | 4.45% | 4.13% | 3.82% | 3.50% | 3.18% | 2.86% | 2.54% | 2.22% |
| Investments | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | |
| | | $ 54.10 | $ 54.10 | $ 54.10 | $ 54.10 | $ 54.10 | $ 54.10 | $ 54.10 | $ 54.10 | $ 54.10 | $ 54.10 |
| | | $(1,054.10) | $ 53.66 | $ 53.66 | $ 53.66 | $ 53.66 | $ 53.66 | $ 53.66 | $ 53.66 | $ 53.66 | $ 53.66 |
| | | | $(1,107.76) | $ 52.86 | $ 52.86 | $ 52.86 | $ 52.86 | $ 52.86 | $ 52.86 | $ 52.86 | $ 52.86 |
| | | | | $(1,160.63) | $ 51.68 | $ 51.68 | $ 51.68 | $ 51.68 | $ 51.68 | $ 51.68 | $ 51.68 |
| | | | | | $(1,212.31) | $ 50.12 | $ 50.12 | $ 50.12 | $ 50.12 | $ 50.12 | $ 50.12 |
| | | | | | | $(1,262.43) | $ 48.16 | $ 48.16 | $ 48.16 | $ 48.16 | $ 48.16 |
| | | | | | | | $(1,310.59) | $ 45.82 | $ 45.82 | $ 45.82 | $ 45.82 |
| | | | | | | | | $(1,356.41) | $ 43.09 | $ 43.09 | $ 43.09 |
| | | | | | | | | | $(1,399.50) | $ 40.00 | $ 40.00 |
| | | | | | | | | | | $(1,439.50) | $ 36.55 |
| | | | | | | | | | | | $12,922.32 |
| PV, FV | $(8,345.28) | | | | | | | | | | $13,398.36 |
| MIRR | 4.85% | | | | | | | | | | |

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Results from Tables 2 to 6 indicate that this simple investment strategy would have resulted in holding period returns ranging from 4.29% (1962-72) to 12.85% (1982-92). The 1982-92 period was concurrent with significant fall in the yield. It dropped sharply after climbing to all-time highs in 1980s (15.84% in July 1981). Interestingly, despite extreme fluctuations in the yield, a long-term bond investor that followed this simple investment strategy never faced negative returns (we account for inflation later). Among the four decades studied, the 1962-72 period had the lowest MIRR of 4.29%.

To further investigate the impact of extreme changes in the yield, in Tables 7 and 8 we simulate the same strategy for 2 consecutive decades of extreme rise and fall in interest rates (July 1971- July 1991). The yield reached its peak in July 1981(15.84%). The strategy would have produced 8.15% return in the 1971-81 period (extreme rise in the yield) and 14.15% return in the 1981-91 period (extreme fall in the yield).

**Table 7:** Annual Contributions of $1000 to Treasury Notes during a Decade Extreme Rise in Interest Rate (July 1971- July 1981 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 6.00% | | | | | | | | | | |
| Incremental Change | 0.98% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 6.00% | 6.98% | 7.97% | 8.95% | 9.94% | 10.92% | 11.90% | 12.89% | 13.87% | 14.86% | 15.84% |
| Investments | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | |
| | | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 |
| | | $(1,060.00) | $ 74.03 | $ 74.03 | $ 74.03 | $ 74.03 | $ 74.03 | $ 74.03 | $ 74.03 | $ 74.03 | $ 74.03 |
| | | | $(1,134.03) | $ 90.36 | $ 90.36 | $ 90.36 | $ 90.36 | $ 90.36 | $ 90.36 | $ 90.36 | $ 90.36 |
| | | | | $(1,224.39) | $ 109.61 | $ 109.61 | $ 109.61 | $ 109.61 | $ 109.61 | $ 109.61 | $ 109.61 |
| | | | | | $(1,334.00) | $ 132.55 | $ 132.55 | $ 132.55 | $ 132.55 | $ 132.55 | $ 132.55 |
| | | | | | | $(1,466.54) | $ 160.15 | $ 160.15 | $ 160.15 | $ 160.15 | $ 160.15 |
| | | | | | | | $(1,626.69) | $ 193.64 | $ 193.64 | $ 193.64 | $ 193.64 |
| | | | | | | | | $(1,820.33) | $ 234.60 | $ 234.60 | $ 234.60 |
| | | | | | | | | | $(2,054.94) | $ 285.06 | $ 285.06 |
| | | | | | | | | | | $(2,340.00) | $ 347.63 |
| | | | | | | | | | | | $13,449.95 |
| PV, FV | $(6,915.01) | | | | | | | | | | $15,137.58 |
| MIRR | 8.15% | | | | | | | | | | |

**Table 8:** Annual Contributions Treasury Notes during a Decade of Extreme Fall in Interest Rate (July 1981- July 1991 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 15.84% | | | | | | | | | | |
| Incremental Change | -0.84% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 15.84% | 15.00% | 14.17% | 13.33% | 12.49% | 11.66% | 10.82% | 9.98% | 9.14% | 8.31% | 7.47% |
| Investments | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | $ (1,000) | |
| | | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 |
| | | $(1,158.40) | $ 173.79 | $ 173.79 | $ 173.79 | $ 173.79 | $ 173.79 | $ 173.79 | $ 173.79 | $ 173.79 | $ 173.79 |
| | | | $(1,332.19) | $ 188.72 | $ 188.72 | $ 188.72 | $ 188.72 | $ 188.72 | $ 188.72 | $ 188.72 | $ 188.72 |
| | | | | $(1,520.91) | $ 202.72 | $ 202.72 | $ 202.72 | $ 202.72 | $ 202.72 | $ 202.72 | $ 202.72 |
| | | | | | $(1,723.64) | $ 215.32 | $ 215.32 | $ 215.32 | $ 215.32 | $ 215.32 | $ 215.32 |
| | | | | | | $(1,938.95) | $ 225.98 | $ 225.98 | $ 225.98 | $ 225.98 | $ 225.98 |
| | | | | | | | $(2,164.94) | $ 234.20 | $ 234.20 | $ 234.20 | $ 234.20 |
| | | | | | | | | $(2,399.14) | $ 239.46 | $ 239.46 | $ 239.46 |
| | | | | | | | | | $(2,638.60) | $ 241.27 | $ 241.27 |
| | | | | | | | | | | $(2,879.87) | $ 239.23 |
| | | | | | | | | | | | $20,921.54 |
| PV, FV | $(6,131.26) | | | | | | | | | | $23,040.64 |
| MIRR | 14.15% | | | | | | | | | | |

Further, we compute MIRR for a different yet simpler investing strategy: a one-time $1000 investment in a 10 year T-notes (instead of annual contributions). In Tables 9 and 10 we simulate this strategy for the same 2 periods (1971-91).

**Table 9:** Initial Investment of $1000 and No Further Contributions during a Decade Extreme Rise in Interest Rate (July 1971- July 1981 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 6.00% | | | | | | | | | | |
| Incremental Change | 0.984% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 6.00% | 6.98% | 7.97% | 8.95% | 9.94% | 10.92% | 11.90% | 12.89% | 13.87% | 14.86% | 15.84% |
| Investments | $ (1,000) | $ - | $ - | $ - | $ - | $ - | $ - | $ - | $ - | $ - | |
| | | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 | $ 60.00 |
| | | $ (60.00) | $ 4.19 | $ 4.19 | $ 4.19 | $ 4.19 | $ 4.19 | $ 4.19 | $ 4.19 | $ 4.19 | $ 4.19 |
| | | | $ (64.19) | $ 5.11 | $ 5.11 | $ 5.11 | $ 5.11 | $ 5.11 | $ 5.11 | $ 5.11 | $ 5.11 |
| | | | | $ (69.31) | $ 6.20 | $ 6.20 | $ 6.20 | $ 6.20 | $ 6.20 | $ 6.20 | $ 6.20 |
| | | | | | $ (75.51) | $ 7.50 | $ 7.50 | $ 7.50 | $ 7.50 | $ 7.50 | $ 7.50 |
| | | | | | | $ (83.01) | $ 9.06 | $ 9.06 | $ 9.06 | $ 9.06 | $ 9.06 |
| | | | | | | | $ (92.08) | $ 10.96 | $ 10.96 | $ 10.96 | $ 10.96 |
| | | | | | | | | $ (103.04) | $ 13.28 | $ 13.28 | $ 13.28 |
| | | | | | | | | | $ (116.32) | $ 16.14 | $ 16.14 |
| | | | | | | | | | | $ (132.45) | $ 19.68 |
| | | | | | | | | | | | $ 1,704.71 |
| PV, FV | $(1,000.00) | | | | | | | | | | $ 1,856.84 |
| MIRR | 6.38% | | | | | | | | | | |

**Table 10:** Initial Investment of $1000 and No Further Contributions during a Decade of Extreme Fall in Interest Rate (July 1981-July 1991 Period)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 15.84% | | | | | | | | | | |
| Incremental Change | -0.837% | | | | | | | | | | |
| Initial Contribution | $ 1,000 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| | | | | | | | | | | | |
| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Yield to Maturity | 15.84% | 15.00% | 14.17% | 13.33% | 12.49% | 11.66% | 10.82% | 9.98% | 9.14% | 8.31% | 7.47% |
| Investments | $ (1,000) | $ - | $ - | $ - | $ - | $ - | $ - | $ - | $ - | $ - | |
| | | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 | $ 158.40 |
| | | $ (158.40) | $ 23.76 | $ 23.76 | $ 23.76 | $ 23.76 | $ 23.76 | $ 23.76 | $ 23.76 | $ 23.76 | $ 23.76 |
| | | | $ (182.16) | $ 25.81 | $ 25.81 | $ 25.81 | $ 25.81 | $ 25.81 | $ 25.81 | $ 25.81 | $ 25.81 |
| | | | | $ (207.97) | $ 27.72 | $ 27.72 | $ 27.72 | $ 27.72 | $ 27.72 | $ 27.72 | $ 27.72 |
| | | | | | $ (235.69) | $ 29.44 | $ 29.44 | $ 29.44 | $ 29.44 | $ 29.44 | $ 29.44 |
| | | | | | | $ (265.13) | $ 30.90 | $ 30.90 | $ 30.90 | $ 30.90 | $ 30.90 |
| | | | | | | | $ (296.03) | $ 32.02 | $ 32.02 | $ 32.02 | $ 32.02 |
| | | | | | | | | $ (328.06) | $ 32.74 | $ 32.74 | $ 32.74 |
| | | | | | | | | | $ (360.80) | $ 32.99 | $ 32.99 |
| | | | | | | | | | | $ (393.79) | $ 32.71 |
| | | | | | | | | | | | $ 3,724.08 |
| PV, FV | $(1,000.00) | | | | | | | | | | $ 4,150.58 |
| MIRR | 15.30% | | | | | | | | | | |

Tables 9 and 10 show that even for a more passive strategy such as a one-time $1000 investment an investor could have earned 6.38% in the 1971-81 period and 15.30% in the 1981-91 period.

Next, we relax one of our initial assumptions. In all previous simulations, we assumed 10-year T-notes make annual coupon payments. This is not the case. All U.S Treasury bonds pay semiannual coupon payments. In Tables 11 and 12 we repeat a similar investment strategy, this time for a 5 year period. We assume our investor makes $500 contributions semiannually for five consecutive years. We compare her holding period returns for these two periods: July 1976- July 1981 and July 1981- July 1996. As was the case before, our investor reinvests the semiannual coupon payments immediately after she receives them. Using a similar methodology, this time with semiannual discounting, we compute MIRR again: 9.37% for the 1976- 81 and 14.68% for the 1981-86 period.

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**Table 11:** Semiannual contributions (July 1976- July 1981)[3]

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 7.55% | | | | | | | | | | |
| Incremental Change | 0.829% | | | | | | | | | | |
| Initial Contribution | $ 500 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| Year | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| Yield to Maturity | 7.55% | 8.38% | 9.21% | 10.04% | 10.87% | 11.70% | 12.52% | 13.35% | 14.18% | 15.01% | 15.84% |
| Investments | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | |
| | | $ 18.88 | $ 18.88 | $ 18.88 | $ 18.88 | $ 18.88 | $ 18.88 | $ 18.88 | $ 18.88 | $ 18.88 | $ 18.88 |
| | | $ (518.88) | $ 21.74 | $ 21.74 | $ 21.74 | $ 21.74 | $ 21.74 | $ 21.74 | $ 21.74 | $ 21.74 | $ 21.74 |
| | | | $ (540.61) | $ 24.89 | $ 24.89 | $ 24.89 | $ 24.89 | $ 24.89 | $ 24.89 | $ 24.89 | $ 24.89 |
| | | | | $ (565.50) | $ 28.38 | $ 28.38 | $ 28.38 | $ 28.38 | $ 28.38 | $ 28.38 | $ 28.38 |
| | | | | | $ (593.88) | $ 32.27 | $ 32.27 | $ 32.27 | $ 32.27 | $ 32.27 | $ 32.27 |
| | | | | | | $ (626.15) | $ 36.61 | $ 36.61 | $ 36.61 | $ 36.61 | $ 36.61 |
| | | | | | | | $ (662.76) | $ 41.50 | $ 41.50 | $ 41.50 | $ 41.50 |
| | | | | | | | | $ (704.26) | $ 47.02 | $ 47.02 | $ 47.02 |
| | | | | | | | | | $ (751.28) | $ 53.27 | $ 53.27 |
| | | | | | | | | | | $ (804.56) | $ 60.39 |
| | | | | | | | | | | | $ 5,921.23 |
| PV, FV | $ (4,017.42) | | | | | | | | | | $ 6,286.18 |
| MIRR | 9.37% | | | | | | | | | | |

**Table 12:** Semiannual contributions (July 1981- July 1986)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Initial Interest Rate | 15.84% | | | | | | | | | | |
| Incremental Change | -0.914% | | | | | | | | | | |
| Initial Contribution | $ 500 | | | | | | | | | | |
| Contribution Growth | 0.00% | | | | | | | | | | |
| Year | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| Yield to Maturity | 15.84% | 14.93% | 14.01% | 13.10% | 12.18% | 11.27% | 10.36% | 9.44% | 8.53% | 7.61% | 6.70% |
| Investments | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | $ (500) | |
| | | $ 39.60 | $ 39.60 | $ 39.60 | $ 39.60 | $ 39.60 | $ 39.60 | $ 39.60 | $ 39.60 | $ 39.60 | $ 39.60 |
| | | $ (539.60) | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 |
| | | | $ (579.87) | $ 40.63 | $ 40.63 | $ 40.63 | $ 40.63 | $ 40.63 | $ 40.63 | $ 40.63 | $ 40.63 |
| | | | | $ (620.50) | $ 40.64 | $ 40.64 | $ 40.64 | $ 40.64 | $ 40.64 | $ 40.64 | $ 40.64 |
| | | | | | $ (661.13) | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 | $ 40.27 |
| | | | | | | $ (701.41) | $ 39.52 | $ 39.52 | $ 39.52 | $ 39.52 | $ 39.52 |
| | | | | | | | $ (740.93) | $ 38.36 | $ 38.36 | $ 38.36 | $ 38.36 |
| | | | | | | | | $ (779.29) | $ 36.79 | $ 36.79 | $ 36.79 |
| | | | | | | | | | $ (816.08) | $ 34.79 | $ 34.79 |
| | | | | | | | | | | $ (850.87) | $ 32.39 |
| | | | | | | | | | | | $ 7,266.44 |
| PV, FV | $ (3,856.34) | | | | | | | | | | $ 7,649.70 |
| MIRR | 14.68% | | | | | | | | | | |

Table 13 summarizes the results. The first panel shows MIRRs for decades of rising rate versus decades of falling rate. The second panel compares holding period returns for 1971-81 and 1981-91 periods in which we observed extreme rise and fall in the yield. Within each group, we also compare the MIRRs for two different strategies: fixed annual contributions versus one time contributions at the beginning of the period. Finally, in the last panel we show MIRRs for fixed semiannual contributions and semiannual coupon payments for two consecutive five-year periods: 1976-81 and 1981- 86. In order to get a sense of inflation-adjust returns, we define Real Return as:

Real Return = MIRR- Inflation

Where the Inflation is the average annualized inflation rate during the period. Results show that our hypothetical investor would have enjoyed significant positive return for 3 out of the 4 decades. Only the 1971-1981 period, which was concurrent with extreme rise of 9.84% in the yield, results in a minor negative return. In this paper, we assumed annual coupon payments so we could simplify the case and at the same time cover the entire decade. Not doing so would have resulted in tables with 20 columns which would have made the calculations more cumbersome.

[3] Here we assumed $500 semiannual contributions to be consistent with $1000 annual contributions in the former Tables. However, the MIRR will remain unchanged, regardless of the contribution dollar amount, as long as the contributions are fixed across the 5 year period. This is also true for the previous Tables.

**Table 13:** Summary Results

| | Period Rate Change (%) | MIRR (%) | Inflation Rate (%)[4] | Real Return (%) |
|---|---|---|---|---|
| **Decades of Rising Rate: Fixed Contributions** | | | | |
| 1962-1972 | 2.26 | 4.29 | 3.10 | 1.19 |
| 1972-1982 | 8.06 | 7.85 | 8.45 | -0.60 |
| **Decades of Falling Rate: Fixed Contributions** | | | | |
| 1982-1992 | -6.64 | 12.85 | 4.13 | 8.72 |
| 1992-2002 | -2.13 | 7.14 | 2.67 | 4.47 |
| 2002-2012 | -3.19 | 4.85 | 2.42 | 2.43 |
| **Decades of Extreme Rise and Fall** | | | | |
| **July 1971- July 1981 (Rise)** | | | | |
| Fixed Annual Contributions | 9.84 | 8.15 | 8.42 | -0.27 |
| One-Time Contribution | 9.84 | 6.38 | 8.42 | -2.04 |
| **July 1981- July 1991 (Fall)** | | | | |
| Fixed Annual Contributions | -8.37 | 14.15 | 4.13 | 10.02 |
| One-Time Contribution | -8.37 | 15.30 | 4.13 | 11.17 |
| **Semiannual Coupon Payments: Fixed Contributions** | | | | |
| July 1976- July 1981 | 8.29 | 9.37 | 9.82 | -0.45 |
| July 1981- July 1986 | -9.14 | 14.68 | 3.81 | 10.87 |

## 3. Conclusion

In this paper we simulate cash in/outflows of an investor that makes fixed periodic contributions to her one-bond portfolio and liquidates her position after a decade (or five year with semiannual coupon payments). Then we calculated her holding-period returns defined as MIRR and hypothesize that she, as a long-term investor, should not be too concerned with interest rate fluctuations for two reasons: first, the dollar cost-averaging strategy mainly used with this simple strategy (which in terms of fixed contributions is similar to contributions to bond funds in retirement accounts) mitigates the risk of fall in portfolio values; second, the opportunity to reinvest ‘guaranteed’ interest payments back in the portfolio helps boosting the holding period returns significantly. Our simulation results indicate that long-term bond investors would have earned positive returns that would have exceeded the inflation rate, regardless of rising or falling interest rates in any particular decade, except for the 1971-81 period during which the interest rate soared by 9.84% . The case studied in this paper has a pedagogical objective as well. Finance students often struggle to fully understand the difference between coupon rate and yield to maturity. Also, the reinvestment rate assumption embedded in the bond pricing model seems difficult to comprehend for most students. By asking students to do similar simulations, they attain a deeper understating of reinvestment rate assumption and yield to maturity. This would help students not only with better understanding of bonds but also with comprehension of related topics such as internal rate of return (IRR) in capital budgeting.

**References**

Light, J., (2013). Time to dump bonds? Maybe not: Investors may be overreacting to the risk of higher interest rates. *The Wall Street Journal*, December 6, 2013.

[4] Inflations were extracted from http://www.usinflationcalculator.com/inflation/historical-inflation-rates/ and http://www.usinflationcalculator.com/