

VOLATILITY MODELLING FOR EURO IN TURKEY

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ABSTRACT

A series is formed by taking the logarithm of Euro exchange rate return between January 3, 2000 and December 12, 2014 so as to analyze the volatility structure of Euro exchange rate return in Turkey. Initially, the series is observed to be a stationary one, and optimal autoregressive moving average model is estimated. Later, it has been revealed that the residuals of the mean equation implicate ARCH effect. In addition, heteroscedastic variance model with four different conditions / assumptions has been estimated. It has been proven that ARCH effect is eliminated in these models estimated and no autocorrelation exists in error terms, either. As a result, it has been revealed that among these models the optimal conditional heteroskedasticity model is the TGARCH (1,1).

Keywords : Euro, Volatility, Heteroskedasticity

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1. INTRODUCTION

Money and financial markets in developing countries are highly susceptible to changes occurring in economy. For example, they are immediately influenced by events like a crisis or a natural disaster that occur in the country. In other words, financial assets are instantly affected by cases of emergency, which brings about fluctuations in them.

The term “volatile” is used to describe the magnitude / size and frequency of the fluctuation occurring generally in the price of a stock, bill or any other kind of financial asset (Güneş ve Saltoğulu, 1998, s.14). Volatility means that a variable shows a dramatic increase or decrease vis a vis a certain mean value (Özden, 2008). Another definition says that volatility is a measure of uncertainty over the returns to be obtained from a financial asset (John Hull, 2006, p.758). Conventional econometric models assume that variance as a measure of volatility does not vary based on time, it is rather considered to be independent from time. Yet, conventional econometric models are not utilized for measuring the volatility since the variances of financial time series somehow vary, generally depending on time. For this reason, using models like ARCH, GARCH, EGARCH and TGARCH instead of conventional econometric models has become the trend.

ARCH, GARCH, EGARCH and TGARCH models are formed in this study, using the logarithmic data of the daily Euro buying rate in Turkey between the dates January 3, 2000 and December 12, 2014. The aim of the study is to identify the optimal model among the ones mentioned.

STATISTICAL MODELS

As mentioned above, models like ARCH, GARCH, EGARCH and TGARCH, which are defined briefly as well, have been used in this study.

1.1. ARCH Model

ARCH model, Autoregressive Conditional Heteroskedasticity, was developed by Engle in 1982 for cases when constant variance assumption was not possible to be obtained in time series. ARMA model can often be denoted as:

$$Y_t = c + \sum_{i=1}^p \theta_i y_{t-1} + \sum_{i=1}^q \phi_i u_{t-1} + u_t$$

It is presumed in ARMA Model that the error terms are distributed as $u_t \sim N[0, (w + \alpha_1 u_{t-1}^2)]$ in (t-1) period. Making use of the denotation given above and under the $w > 0$, $\alpha_i \geq 0$ ve $\sum_{i=1}^q \alpha_i < 1$ assumption, ARCH Model is obtained as:

$$h_t = w + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2$$

It is important to explore whether ARCH effect applies or not so that the ARCH model expressed above can be applied. The test intending to find out whether autoregressive conditional heteroskedastic effects exist or not in time series was developed by Engle in 1982. It is possible to find out if ARCH effect applies or not through LM and Q tests. The former is often the one opted for in practice. LM statistic has χ^2_q distribution, and is calculated as:

$$LM = (T-p)R^2$$

If the calculated value proves to be less than the χ^2 table value, $H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_q$ hypothesis is accepted, and it is determined that ARCH effect does not exist. In a reverse case, H_0 hypothesis is rejected, and it is determined that ARCH effect exists (Kökçen, 2010).

1.2.GARCH Model

Having enhanced ARCH model, Bollerslev (1986) put forth the GARCH model which is the generalized ARCH model. The most convenient model used for volatility estimation in practice is the GARCH model.

When the lag length of error squares and lag length of autoregressive section are described by q and p, respectively a general GARCH (p,q) model is obtained (Bollerslev,1987). However, GARCH model is denoted as $h_t = w + \sum_{j=1}^p \beta_j h_{t-j} + \sum_{i=1}^q \alpha_i u_{t-i}^2$ since $w > 0$, $\alpha_i \geq 0$ ve $\beta_j > 0$ assumptions are accepted in the model. As in ARCH model, it is required to calculate the GARCH effect in the GARCH model formed. LM test is also applied, with a few slight changes, for the calculation of GARCH effect. Test statistic calculated as TR^2 aligns with χ^2 distribution with p+q degrees of freedom (Kutlar, 2005). If the LM statistic proves to be greater than the table value, it is concluded that GARCH effect is significant.

1.3.EGARCH Model

In analyses carried out through GARCH Models, volatility is assumed to yield a symmetrical effect against negative and positive shocks so it is not possible to calculate the leverage effect in analyses regarding financial time series. Upon this, EGARCH models were developed by Nelson for the first time as the exponential GARCH model. EGARCH model is the exponential generalized autoregressive conditional heteroskedasticity model.

EGARCH model is denoted as follows:

$$\text{Log} (h_t) = w + \sum_{j=1}^p \beta_j \log(h_{t-j}) + \sum_{i=1}^q \alpha_i \frac{|u_{t-i}|}{\sqrt{h_{t-i}}} + \sum_{i=1}^q \gamma_i \frac{u_{t-i}}{\sqrt{h_{t-i}}}$$

$\gamma_i > 0$ seen in the model shows that skewness / asymmetry exists. $\gamma_i < 0$ represents the existence of leverage effect, which means that the effect of negative shocks (with the same size) on volatility is more than that of positive shocks.

1.4.TGARCH Model

TGARCH is another alternative model which considers the fact that the effect of positive shocks and negative shocks are not symmetrical (Threshold GARCH). TGARCH models is based on the assumption that when $u_{t-1} = 0$ is regarded as the threshold value, the effect of positive news (positive shocks, $u_{t-1} > 0$) will be less than the effect of negative news (negative shocks, $u_{t-1} < 0$) on the conditional variance (Mapa, 2004:3-5). This effect is incorporated into TGARCH model by including D_{t-1} dummy variable in the model. Based on this, TGARCH(p,q) model is denoted as follows:

$$h_t = w + \sum_{j=1}^p \beta_j h_{t-j} + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \sum_{i=1}^q \gamma_i D_{t-1} u_{t-i}^2$$

$$D_{t-1} = \begin{cases} 1 & u_{t-1} < 0, \\ 0 & u_{t-1} \geq 0, \end{cases}$$

Given that $\gamma_i \neq 0$ in such a model, it is said that the effect of the recent news will be different. When the effect of positive news is as much as α_i , that of negative news will correspond to $\gamma_i + \alpha_i$. $\gamma_i > 0$ refers to the fact that the effect of negative news on volatility will be greater than that of positive news. By this, it is the leverage effect from i^{th} level. On the other hand, given that $\gamma_i = 0$, it is understood that the effect of recent news on volatility is not asymmetrical. In such a case, TGARCH model will be equal to GARCH model (Hossain vd., 2005:419-425).

2. DATA AND FINDINGS

For the modelling of Euro exchange rate return in Turkey, daily Euro exchange rate returns dating from January 3, 2000 to December 12, 2014 are calculated. Obtained from the website of the Central Bank of Turkey, the relevant data comprises only the days when transactions were available. $\ln(\text{Euro}_t / \text{Euro}_{t-1})$ formula has been obtained. As one can see from the formula, the logarithm of the series has been used for operational convenience. Exchange rate values prior to December 31, 2004 are integrated into the operations by dividing the daily exchange rate values into one million since 6 zeros were removed from the Turkish lira as of December 31, 2004.

Euro return values are made up of values of 3762 days. It is clearly seen in the figure below that maximum and minimum return values are close to 0. In addition, with leptokurtosis level being 208, it is seen that the series has a high level of leptokurtosis. The mean and standard deviation values being close to 0 shows that the series may be a stationary one.

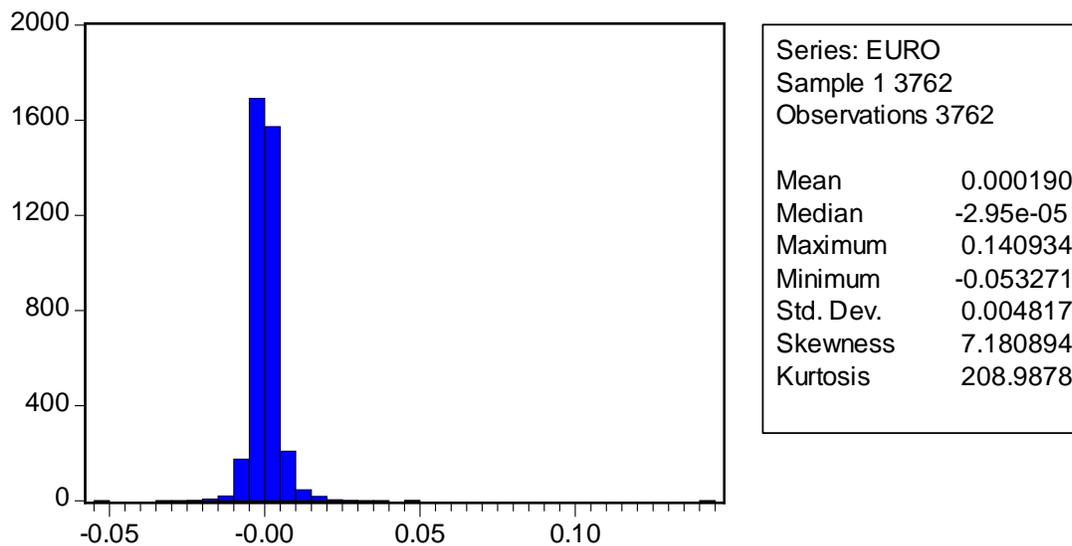


Figure 1 : Euro logarithmic return characteristic values

Being stationary is an important concept regarding time series. The graph of the series is given below. Even before the stationarity test is applied, it can be said that the series has a stationary structure by having a glance at the graph of the series. It can also be obviously seen that the series has a volatile pattern. Such series are known to have a high level of volatility, particularly in finance.

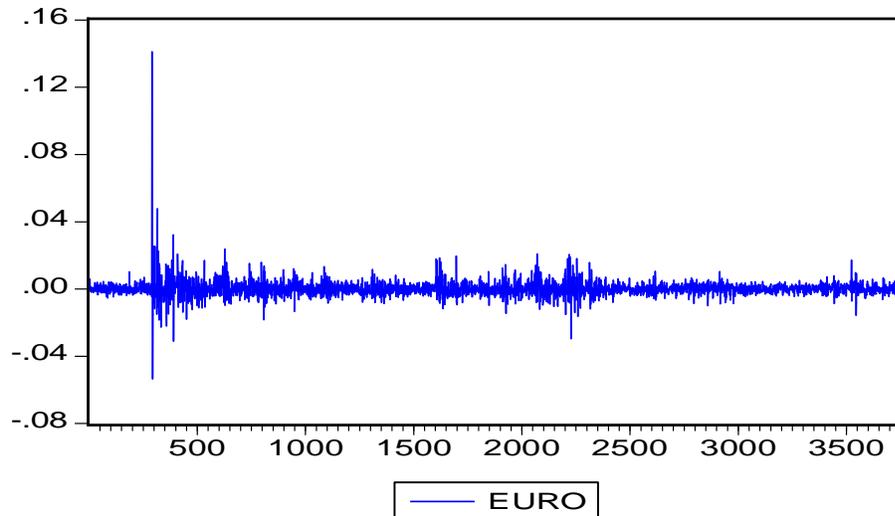


Figure 2 : Euro logarithmic return values

As can be seen from Table 1, the P value is 0.0001 at a significance level of 5 % when stationarity test is applied for Euro return series, with the help of Augmented Dickey-Fuller Test. It is anticipated that there exists no unit root and the series is stationary since the P value is less than 0.05. If the series were not stationary, it would be required to render it stationary. However, while calculating the return series, the difference is already taken. For this reason, such series is generally considered to be stationary.

Table 1: Augmented Dickey-Fuller Unit Root Test

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-48.85939	0.0001
Test critical values:		
1% level	-3.431903	
5% level	-2.862112	
10% level	-2.567118	

It is required to estimate the optimal autoregressive moving average model following the testing of stationarity. For the estimation, the Least Squares Method has been used. Among the different models tested, the best one has proved to be AR(1). Table 2 shows that the model and parameters are significant.

Table 2: AR(1) Model for Return Series

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000190	8.66E-05	2.195127	0.0282
AR(1)	0.097469	0.016233	6.004347	0.0000

Following this stage, models used for volatility are presented. However, it is important to know whether there is an ARCH effect on the errors of conditional mean equation or not. ARCH LM test has been performed for this purpose. Table 3 shows the test results that prove the existence of the ARCH effect. As the P-value is less than 0.05, ARCH effect applies. In addition, ARCH effect should disappear once modelling is completed. After the models are identified, it is required to test whether ARCH effect exists or not.

Table 3: ARCH LM Test

F-statistic	26.92069	Prob. F(1,3758)	0.000000
Obs*R-squared	26.74344	Prob. Chi-Square(1)	0.000000

ARCH and GARCH models are the most frequently used structures in volatility estimation. Yet, these models exhibit only symmetrical effect on the return volatility. For this reason, this study also examines TGARCH and EGARCH models since they exhibit asymmetrical effect.

ARCH(1) , GARCH(1,1) , EGARCH(1,1) and TGARCH(1,1) have been found out to be the best ones among the heteroskedastic variance models tried.

The following models are formed for the following conditions:

$$\text{For ARCH(1) ; } h_t = \text{Var}(u_{tq}) = \sigma_t^2 = -0.000337 + 0.236925u_{t-1}^2 \quad (1)$$

$$\text{For GARCH(1,1) ; } h'_t = 7.44E - 07 + 0.6326u_{t-1}^2 + 0.5967h'_{t-1} \quad (2)$$

$$\text{For EGARCH(1,1) ; } \log(h''_t) = -1.4772 + 0.6525 \frac{|u_{t-1}|}{\sqrt{h''_{t-1}}} + 0.1184 \frac{u_{t-1}}{\sqrt{h''_{t-1}}} + 0.9067 \log(h''_{t-1}) \quad (3)$$

$$\text{For TGARCH(1,1) ; } h'''_t = 1.34E - 06 + 1.2231u_{t-1}^2 - 0.9217u_{t-1}^2 D_{t-1} + 0.5117h'''_{t-1} \quad (4)$$

Furthermore, the P-value of all the parameters is 0.00, which demonstrates that they are significant.

ARCH(1) and GARCH(1,1) models have a symmetrical effect on return volatility while EGARCH(1,1) and TGARCH(1,1) models have an asymmetrical effect. When this proposition is considered, it is possible to be informed about whether the model to be chosen among the four models will have a symmetrical or an asymmetrical effect.

Information criteria are utilized during the stage of identifying the optimal conditional heteroskedastic variance model. The values regarding the relevant information criteria are presented in Table 4. The information criterion value that has the least numerical value is considered to be the best model at this point. Moreover, ARCH models have been applied as ARCH effect exists. Table 4 depicts whether ARCH effect is eliminated or not after these models have been applied.

Table 4: Results of Autoregressive Conditional Heteroskedasticity Model

	ARCH(1)	GARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)
Akaike info criterion	-8.110282	-8.323057	-8.306502	-8.338437
Schwarz criterion	-8.103653	-8.314771	-8.296559	-8.328495
Log likelihood	15255.39	15656.51	15626.38	15686.43
Arch LM - P	0.805	0.889	0.865	0.917
Best Model	4	2	3	1

As can be seen in Table 4, ARCH effect disappears since the P-values are greater than 0.05 in ARCH LM test. It seems apparent that none of the four models have ARCH effect. TGARCH(1,1) model is considered to be the best one that has the least value based on Akaike and Schwarz information criteria or the highest value based on Log likelihood. In other words, TGARCH(1,1) proves to be the model that provides the best explanation of the volatility of Euro logarithmic returns. Besides this, shocks have an asymmetrical effect on the Euro logarithmic return volatility, and it has been found out that autocorrelation does not exist in the error terms of the models, either.

3. CONCLUSION

Linear models have a high power of estimation thanks to their statistical structure, and are used in various areas. Yet, non-linear models are also seen in many different phenomena. The use of non-linear models is another major concern since their estimation and modelling is more difficult owing to their structure compared to linear model. For example, it is not possible to estimate volatility through linear models due to heteroskedastic variance. Therefore, it would be more appropriate to use non-linear models for volatility which has significance regarding its estimation since it is confronted mainly in financial series, and money is at stake, accordingly. Such models also give the opportunity to model the heteroskedastic variance.

In this study, however, volatility modelling has been attempted for logarithmic returns regarding Euro exchange rate in Turkey. Heteroskedastic variance model with four different conditions have been used, and significant models have been selected among them. TGARCH(1,1) has proven to be the one that provides the best explanation. The fact that shocks have asymmetrical effect and volatility has an asymmetrical shock wave can be seen from TGARCH(1,1) model as well. The main point herein is that Euro has always been on the rise since it went into circulation in Turkey, which is somehow a proof of its asymmetrical structure.

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