

PORTFOLIO MANAGEMENT: MEAN-VARIANCE ANALYSIS IN THE US ASSET MARKET

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ABSTRACT

In practice an investor would like to have the highest return possible. However, assets with high return usually correlate with high risk. The expected return and risk measured by the variance are the two main characteristics of a portfolio. Mean-variance model as a good optimizer can exploit the correlation, the expected return, and the risk and user constraints to obtain an optimized portfolio. Although it is the simplest model of investments, is sufficiently rich to be directly useful in applied problems and decision theory.

In the paper two methods are presented that exemplify the flexibility of its application: maximizing the return and minimizing the risk. Based on the numerical solution it can easily be understood that, for an investor, specifying a target return may be more intuitive than struggling with risk aversion coefficient λ . The problems are formulated by using the Wolfram Mathematical Programming System.

Key words: portfolio optimization, mean-variance model and decision theory.

1. Introduction

In practice an investor would like to have the highest return possible. However, assets with high return usually correlate with high risk. The expected return and risk measured by the variance are the two main characteristics of a portfolio. The behaviour of portfolio can be quite different from the behaviour of individual components of the portfolio. The risk of a constructed portfolio could be half the sum of the risk of individual assets in the portfolio. This is due to complex correlation patterns between individual assets. Therefore, we need a portfolio optimization.

Mean-variance model as a good optimizer can exploit the correlation, the expected return, and the risk and user constraints to obtain an optimized portfolio. Although it is the simplest model of investments, is sufficiently rich to be directly useful in applied problems and decision theory.

Since Markowitz published his seminal works on mean-variance portfolio selection in 1952 and 1968, his classical model has been serving as a basic of modern finance theory. The mathematical formulation of the Markowitz's portfolio selection problem is the trade-off between risk and return which combines probability theory and optimization theory to model the behaviour of the economic agent. This is the first quantitative treatment of the theory and we cannot call it Modern Portfolio Theory (MPT) as some people, and use it as a main workforce on which analytical portfolio management is based. We should be very careful because this classical model is valid if the return is multi-variate normally distributed and the investor is averse to risk (prefers lower risk), or if for any given return which is multi-variate distributed the investor has quadratic objective function.

As Britten-Jones (1999) notes: "Mean-Variance analysis is important for both practitioners and researchers in finance. For practitioners, theory suggests that mean-variance efficient portfolios can play an important role in portfolio management application. For researchers in finance mean-variance analysis is central to many asset pricing theories as well as to empirical tests of those theories; however practitioners have reported difficulties in implementing mean-variance analysis."

The difficulties in implementing this analysis in practice mainly arise as a result of the following four assumptions:

- Single-period model. Meaning, investors can only make decisions at the beginning and must wait for the results without adjusting the portfolio weights until the end of the horizon.
- Preferences depend only on the mean and variance of payoffs
 - At a given mean, lower variance is preferred
 - At a given variance, a higher mean is preferred.
- Price-taking with no taxes or transaction costs (known as the assumption of "perfect capital markets")
- Investors have enough historical data and that the situation of asset market in future can be correctly reflected by asset data in the past. But what if new assets are listed in the market, there is no past information for these securities.

However, the main goal behind the concept of portfolio management is to combine various assets into portfolios and then to manage those portfolios so as to achieve the desired investment objectives. To be more specific, the investors needs are mostly defined in terms of profit and risk, and the portfolio manager makes a sound decision aimed either to maximize the return or minimize the risk. For further details we refer an interested reader to Estrada (2006, 2008), Ballesteros(2005), Deng et.al.(2005), Grootveld and Hallebach(1999), Nawrocki (1999), and Hallow (1991).

2. Markowitz Mean-Variance Portfolio Theory

In Markowitz Mean-Variance Portfolio Theory the rate of return of assets are random variables. The goal is than to choose the portfolio weighting factors optimally. Meaning, the investor's portfolio achieves an acceptable expected rate of return with minimal volatility. Here as a surrogate for the volatility is taken the variance of the rate of return.

Let us now consider constructing a portfolio consisting of n assets. We have an initial budget x_0 that we wish to assign. The amount that we assign to asset i is $x_{0i}=w_i x_0$ for $i=1,2,\dots,n$, where w_i is weighting factor for asset i . What is also important to note, we allow the weights to take negative values. If a negative values accure, the asset is being shorted in the portfolio. Therefore, to preserve the budget constraint we require that the weights sum to be 1, $\sum_{i=1}^n w_i = 1$. Thus, the sum of the investment is,

$$\sum_{i=1}^n w_i x_0 = x_0 \quad \sum_{i=1}^n w_i = 1$$

The returns are also dependent on each other in a certain way and the dependence will be described by the covariance between them. The covariance between the i -th and the j -th return is denoted by,

$$\sigma_{ij} = \text{cov}(X_i, X_j) = E(X_i - \mu_i)(X_j - \mu_j)$$

Note that in this notation, σ_{ij} stands for the variance of the return of i -th asset,

$$\sigma_{ij} = E(X_i - \mu_i)^2$$

Meaning, given a universe of assets with random return R_i , $i = 1, \dots, n$, expected return μ_i , and covariance matrix $\Sigma = [\sigma_{ij}]$, we should select portfolio weights w_i adding up to one. Given a vector of weights representing the portfolio, its expected return and variance are given as follows:

$$\mu_p = \sum_{i=1}^n \mu_i w_i = \mu^T w, \quad \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i \sigma_{ij} w_j = w^T \Sigma w \quad \text{Eq.1}$$

We continue the analysis by describing the set of all optimal portfolios known as the mean variance efficient portfolios. The standard deviation of return, σ_p might be considered as a risk measure, as it provides us with a measure of dispersion. Actually, variance and standard deviation are interchangeable notions in this case.

Then, we may trade off expected return (μ_p) against risk (σ_p). Of course the trade off may be unclear, but it can be visualized by tracing the frontier of mean- variance efficient portfolios, depicted in Fig. 1.

A portfolio is efficient if it is not possible to obtain a higher expected return without increasing risk or, seeing things the other way around, if it is not possible to decrease risk without decreasing expected return.

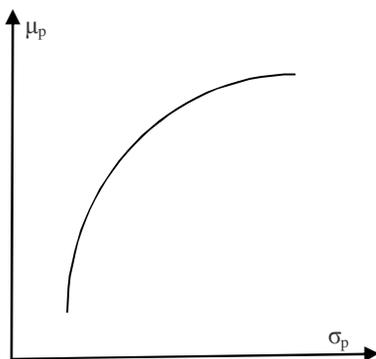


Figure 1. Mean- variance efficient portfolio frontier

Now we are in position to state the optimization problem. The optimization problem behind the first formulation of mean-variance analysis is,

$$\begin{aligned}
 1. \quad & \mathbf{min} \quad w^T \Sigma w \\
 & \mathbf{s.t.} \quad w^T \mu = \mu_i \\
 & \quad \quad \sum_{i=1}^n w_i = 1 \\
 & \quad \quad w_i \geq 0
 \end{aligned}$$

Similarly, the optimization problem behind the second formulation is,

$$\begin{aligned}
 2. \quad & \mathbf{max} \quad \mu^T w - \frac{\lambda}{2} w^T \Sigma w \\
 & \mathbf{s.t.} \quad \sum_{i=1}^n w_i = 1 \\
 & \quad \quad w_i \geq 0
 \end{aligned}$$

3. Application

3.1. Data

The data used is obtained on the basis of *Independent Investment Research, Morningstar* (available online: <http://www.morningstar.com/>). We consider a sample of 8 assets with their monthly history of return for the period January 2012 - June 2014. The portfolio is composed of highly risky assets and less risky assets. Less risky assets are the following:

1. American Funds US Government Sec A AMUSX.
2. American Funds New Economy A ANEFX.

The remaining 6 assets listed below, are with higher risk.

3. USAA Mutual Fund Trust Precious Metals & Minerals Fund USAGX.
4. American Funds Bond Fund of Amer A BFABX.
5. T Rowe Price Equity Fund PRFDX.
6. Fairholme Fund FAIRX.
7. Dodge & Cox Stock Fund DODGX.
8. Fidelity Contra Fund FCNTX.

3.2. Numerical Results

III.2.1. Risk and Return by asset

Using Wolfram Mathematical Programming System we generate the following results:

Table1. Risk and Return			
	Assets	Risk (Variance)	Returns (Mean)
1.	Nv1	0.498177	0.0876667
2.	Nv2	10.4806	2.12667
3.	Nv3	99.4055	-1.451
4.	Nv4	0.634793	0.17
5.	Nv5	6.63884	1.642
6.	Nv6	27.358	2.47133
7.	Nv7	8.91953	2.088
8.	Nv8	8.70336	1.67967

Note: Equation 1 is used to calculate the risk and return of every each asset.

As we can notice in Table 1, the USAGX asset is with variance 99.4055 and mean -1.451. Based on this information most of the investors will decide not to consider the asset 3 in their portfolio. But before drawing any final decision we should take in consideration the possible correlation between asset with the lowest risk (asset 1) and the asset with highest risk (asset 3) .If the returns are negatively correlated ($\sigma < 0$) including asset 1 may, in fact, be beneficial in reducing risk. In our case the correlation is -0.681667. Meaning, the asset 1 will be included in the portfolio.

III.2.2. Mean-Variance model

The main goal behind the concept of mean-variance model is to combine various assets into portfolios and then to manage those portfolios so as to achieve the desired investment objectives. To be more specific, there are two main possibilities to make a sound decision: either to minimize the risk or maximize the return.

- **Quadratic programming model for portfolio optimization by risk factor 1 (maximize the return)**

$$\begin{aligned} \max \quad & \mu^T w - \frac{\lambda}{2} w^T \Sigma w \\ \text{s.t.} \quad & \sum_{i=1}^n w_i = 1 \\ & w_i \geq 0 \end{aligned}$$

From a mathematical perspective, this is a Quadratic programming model. Coefficient λ can be interpreted as a risk aversion coefficient. By changing the value of λ , we can trace the efficient set. Unfortunately, it is a bit difficult to get a feeling for the parameter λ . Anyway, common sense and experience suggest risk aversion coefficients λ in the range between 2 and 4. Therefore, we calculate the weights for three scenarios where λ is 2, 3 and 4 (see Table 2).

weights	$\lambda=2$	$\lambda=3$	$\lambda=4$
W1	0.800184	0.83133	0.846745
W2	0	0	0.0000157468
W3	0	0	0
W4	0.0000128686	0.0000262603	0.000068657
W5	0.0000252394	0.000123988	0.00104384
W6	0	0	0.0000405608
W7	0.199763	0.168497	0.152071
W8	0	0	0.0000135973
Max Return	0.0164064	-0.203487	-0.407919

Note:

- if λ is 2, maximum return is 0, 016% and we should invest in asset 1,4,5 and 7 in order to optimize the risk of the portfolio.
- If λ is 3 or 4 we can notice that the return is negative. By increasing λ the loss will be higher (from -0,203487 to -0,407919).

- **Quadratic programming model for portfolio optimization: constraint approach (minimize the risk)**

$$\begin{aligned} \min \quad & w^T \Sigma w \\ \text{s.t.} \quad & w^T \mu = \mu_i \\ & \sum_{i=1}^n w_i = 1 \\ & w_i \geq 0 \end{aligned}$$

Targeting the return to 7% and 8% we calculate the asset weights and risk (see Table 3). Accordingly, the investor can make an intelligent decision in which assets to invest as well as their weights.

weights	Target return is 7 %	Target return is 8%
W1	0	0
W2	1.19688	1.20781
W3	0	0
W4	0	0
W5	0	0
W6	1.0627	1.553
W7	0.000184011	0
W8	0	0
Min Risk	176.212	248.461

Note:

- **If we specify the return to 7% then the minimum risk is 176.212 and we should invest in asset 2, 6 and 7.**
- **If we specify the return to 10% then the minimum risk will be 248.461, and we should invest in asset 2 and 6.**

4. Conclusion

Due to complex correlation patterns between individual assets, portfolio optimization is a key idea in investing. Mean-variance model as a good optimizer can exploit the correlation, the expected return, and the risk and user constraints to obtain an optimized portfolio. Therefore, application of Mean-Variance model is quite common and one of standard tools of decision making in both theory and practice. But we should be very careful because this classical model is valid if the return is multi-variates normally distributed and the investor is averse to risk (prefers lower risk), or if for any given return which is multi-variates distributed the investor has quadratic objective function.

In the paper two methods are presented that exemplify the flexibility of its application: maximizing the return and minimizing the risk. Based on the numerical solution provided in section 3, it can easily be understood that, for an investor, specifying a target return may be more intuitive than struggling with risk aversion coefficient λ .

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